CERN–TH/95-300 – hep-th/9511108
CPTH–S396.1195 – LPTENS–95/49

$N = 2$ Heterotic Superstring and its Dual Theory in Five Dimensions[⋆]

I. Antoniadis[a], S. Ferrara[b] and T.R. Taylor[c,d]

[a]*Centre de Physique Théorique, Ecole Polytechnique,*[†] *F-91128 Palaiseau, France*

[b]*Theory Division, CERN, 1211 Geneva 23, Switzerland*

[c]*Laboratoire de Physique Théorique de l'Ecole Normale Supérieure,*[♮]
24, rue Lhomond, F-75231 Paris, France

[d]*Department of Physics, Northeastern University, Boston, MA 02115, U.S.A.*

Abstract

We study quantum effects in five dimensions in heterotic superstring theory compactified on $K_3 \times S_1$ and analyze the conjecture that its dual effective theory is eleven-dimensional supergravity compactified on a Calabi-Yau threefold. This theory is also equivalent to type II superstring theory compactified on the same Calabi-Yau manifold, in an appropriate large volume limit. In this limit the conifold singularity disappears and is replaced by a singularity associated to enhanced gauge symmetries, as naïvely expected from the heterotic description. Furthermore, we exhibit the existence of additional massless states which appear in the strong coupling regime of the heterotic theory and are related to a different type of singular points on Calabi-Yau threefolds.

[⋆]Research supported in part by the National Science Foundation under grant PHY–93–06906, in part by the EEC contracts SC1–CT92–0792, CHRX-CT93-0340 and SC1*CT92–0789, in part by the U.S. DOE grant DE-FG03-91ER40662 Task C, and in part by CNRS–NSF grant INT–92–16146.

[†]Laboratoire Propre du CNRS UPR A.0014.

[♮]Unité Propre du CNRS, associée à l'Ecole Normale Supérieure et à l'Université de Paris-Sud.

November 1995

1. Introduction

Among different duality conjectures, one of the most appealing is the possible relation of superstring theory with an underlying 11-D theory which may have only a non-perturbative definition [1]. There is an evidence that this theory, whose low-energy limit is described by the coupling constant-free 11-D supergravity, must be suitably implemented by two- and five-brane solitonic solutions, in order to be at least equivalent to lower dimensional string theories [2]. These solutions imply that in seven dimensions the heterotic string compactified on T_3 should be equivalent to 11-D supergravity compactified on K_3 [1] and that in five dimensions the heterotic string compactified on $K_3 \times S_1$ should be equivalent to 11-D supergravity compactified on a Calabi-Yau threefold [3, 4]. These compactifications preserve $N = 2$ simple supersymmetry in $D = 7$ and in $D = 5$, respectively.[1]

The interest in dealing with a five-dimensional theory is that, unlike in higher dimensions, the spectrum of massless states can change model by model. The numbers of neutral massless vector multiplets and hypermultiplets of the heterotic string, in the abelian phase, should be in correspondence with the Hodge numbers $h_{(1,1)}$ and $h_{(2,1)}$ of the 11-D theory [5, 6]. Indeed, in $D = 5$, the number of vector multiplets is $n_V = h_{(1,1)} - 1$ and the number of hypermultiplets $n_H = h_{(2,1)} + 1$. The simplest model must have $h_{(1,1)} = 3$ since on the heterotic side, compactified on $K_3 \times S_1$, one gets at least three vector fields: $g_{\mu 6}$, $b_{\mu 6}$ and the antisymmetric tensor $b_{\mu\nu}$ which is dual to a vector in $D = 5$. One of the vector bosons corresponds to the graviphoton of the gravitational supermultiplet while the remaining two form vector multiplets whose (real) scalar components are the 5-D dilaton ϕ and the radius field R associated with S_1. A possible way to reduce the number of vectors is to consider a 5-D theory with a fixed radius R_0. In that case only one combination of $g_{\mu 6}$, $b_{\mu 6}$ appears which corresponds to the graviphoton, and the theory is related to $h_{(1,1)} = 2$ Calabi Yau threefold. In general if a gauge group of rank r is added so that $h_{(1,1)} = 2 + r$, the scalar

[1]We call simple the smallest possible supersymmetry existing in a given dimension D.

components of vector multiplets parameterize the space

$$\mathcal{M} = O(1,1) \times \frac{O(1,r)}{O(r)}. \tag{1.1}$$

It is well known that, in five dimensions, the $(h_{(1,1)}-1)$-dimensional space $\mathcal{M}$ of scalar components of $N = 2$ abelian vector multiplets coupled to supergravity can be regarded as a hypersurface of a $h_{(1,1)}$-dimensional manifold whose coordinates X are in correspondence with the vector bosons, including the graviphoton [7]. The equation of the hypersurface is $\mathcal{V}(X) = 1$, where $\mathcal{V}$ is a homogeneous cubic polynomial in X's. If $\mathcal{V}$ is factorizable, *i.e.* of the form $\mathcal{V} = sQ(t)$, where Q is a quadratic form in t's, then $\mathcal{M}$ is given precisely by (1.1).

It is also known that the function $\mathcal{V}(X)$ describing the vector multiplet sector of 11-D supergravity compactified on a Calabi-Yau threefold (CY) is given by the CY intersection form [8, 6, 3]. If we consider a threefold obtained by a K_3 fibration [9], then

$$\mathcal{V} = sQ(t) + C(t) \ , \tag{1.2}$$

where $Q(t)$ is quadratic and $C(t)$ is cubic. From the heterotic point of view, identifying s as the inverse of the string loop expansion parameter ($s \sim 1/g^2$), we see that the first term corresponds to a tree-level contribution and gives the desired scalar manifold (1.1). Hence K_3 fibrations provide good candidates for CY duals of the heterotic theory. The cubic form $C(t)$ should be obtained by a one-loop calculation in the heterotic theory, reproducing all remaining CY intersection numbers. In this paper we show that this is indeed the case.

The paper is organized as follows. In section 2, we discuss the basic features of 5-D theories obtained by compactifying 11-D supergravity on Calabi-Yau threefolds. For CY manifolds which are K_3 fibrations, we perform a duality transformation which brings the lagrangian to a form that can be compared with the 5-D heterotic superstring, with one of the vector bosons replaced by the antisymmetric tensor field. In section 3, we determine the one-loop effective action by computing appropriate heterotic superstring amplitudes in a rank 2+1 model. Our results agree with the known form of the 4-D $N = 2$ prepotential

of the heterotic theory compactified on $K_3 \times T_2$ [13, 14], in the decompactification limit in which one of the torus radii goes to infinity. We argue that the one-loop results are exact, at least in some finite region of the moduli space. In section 4, we discuss duality between the heterotic theory and 11-D supergravity compactified on Calabi-Yau threefolds, providing a dual description of the enhanced gauge symmetry points which unlike in four dimensions remain present in the full quantum theory.[2] In section 5, we exhibit the existence of additional massless states which appear in the strong coupling regime of the heterotic theory.

2. 11-D Supergravity Compactified to $D = 5$ on Calabi-Yau Threefolds

In this section we will recall the basics of the low-energy theory of 11-D supergravity compactified on a generic Calabi-Yau threefold with Hodge numbers $h_{(1,1)}$, $h_{(2,1)}$ and the intersection numbers $C_{\Lambda\Sigma\Delta}$ $(\Lambda, \Sigma, \Delta = 1, \ldots, h_{(1,1)})$ [3]. The bosonic fields of 11-D theory are the elfbein $e_{\hat{\mu}\hat{a}}$ and the three-form gauge field $\mathcal{A}_{\hat{\mu}\hat{\nu}\hat{\rho}}$, with all indices running over $1, \ldots, 11$. It is convenient to split these indices as $\hat{\mu} = (\mu, i, \bar{\jmath})$, $\mu = 1, \ldots, 5$ and $i, \bar{\jmath} = 1, 2, 3$. The $h_{(1,1)}$ moduli split then into $h_{(1,1)} - 1$ moduli with unit volume ($\det g_{i\bar{\jmath}} = 1$) and the volume modulus $\det g_{i\bar{\jmath}}$. The massless spectrum contains $h_{(1,1)} - 1$ vector multiplets with real scalar components given by the moduli at unit volume. The vector bosons, including the graviphoton, are the $h_{(1,1)}$ one-forms (on space-time) $\mathcal{A}_{\mu i\bar{\jmath}}$. There is one universal hypermultiplet with the scalar components $(\det g_{i\bar{\jmath}}, \mathcal{A}_{\mu\nu\rho}, \mathcal{A}_{ijk} = \epsilon_{ijk} a)$. There are also $h_{(2,1)}$ additional hypermultiplets whose scalar components are given by the complex scalar pairs $(g_{i\bar{\jmath}}, \mathcal{A}_{ij\bar{k}})$. For our purposes, the most important fact is the absence from the spectrum of a scalar corresponding to a two-index antisymmetric tensor field with both internal indices.

[2]Enhanced gauge symmetries on CY manifolds may also appear in some cases in $D = 4$ as recently discussed in refs.[10, 11].

The absence of such a field implies that there are no non-perturbative instanton corrections to the low-energy effective action describing the vector multiplet sector of the theory [16].

The effective $N = 2$ supersymmetric lagrangian describing vector multiplets coupled to supergravity is completely determined by one function $\mathcal{V}(X)$, a homogeneous cubic polynomial of the vector coordinates X^Λ [7]. As already mentioned before, in the case of a theory obtained by compactifying 11-D supergravity on a Calabi-Yau threefold, this function is given by the intersection form [3]:[3]

$$\mathcal{V} = \frac{1}{6} C_{\Lambda\Sigma\Delta} X^\Lambda X^\Sigma X^\Delta \tag{2.1}$$

The bosonic part of the lagrangian is given then by [7]

$$\mathcal{L}_{\mathrm{b}} = -\frac{1}{2}\mathcal{R} - \frac{1}{2} g_{xy} \partial\phi^x \partial\phi^y - \frac{1}{4} G_{\Lambda\Sigma} F^\Lambda F^\Sigma + \frac{1}{48} C_{\Lambda\Sigma\Delta} \epsilon F^\Lambda F^\Sigma A^\Delta \ , \tag{2.2}$$

where $\mathcal{R}$ is the Ricci scalar and in the last term the space-time indices are contracted by using the completely antisymmetric 5-D ϵ-tensor. The scalars parameterize the hypersurface $\mathcal{V}(X) = 1$, and their metric is related to the vector metric by

$$g_{xy} = G_{\Lambda\Sigma}\, \partial_{\phi_x} X^\Lambda \partial_{\phi_y} X^\Sigma \Big|_{\mathcal{V}=1} . \tag{2.3}$$

Finally, the vector metric

$$G_{\Lambda\Sigma} = -\frac{1}{2}\, \partial_\Lambda \partial_\Sigma \ln \mathcal{V}|_{\mathcal{V}=1} . \tag{2.4}$$

For a generic Calabi-Yau manifold there is no preferred vector field, however in the case of manifolds obtained by K_3 fibrations, the form (1.2) of the function $\mathcal{V}$ singles out s. We will dualize the gauge vector field A^s_μ into an antisymmetric tensor and identify it with the $b_{\mu\nu}$ field of the dual heterotic string theory. The scalar manifold will be parameterized by t_k with $k = 1, \ldots, h_{(1,1)} - 1$; s will be eliminated by using the constraint

$$\mathcal{V} = 1 \ \Rightarrow\ s = \frac{1 - C(t)}{Q(t)} \ . \tag{2.5}$$

[3] This corresponds to the “very special geometry” of ref.[12].

Using eq.(2.4) with the constraint (2.5) one obtains

$$
\begin{aligned}
G_{ss} &= \frac{1}{2}Q^2 \\
G_{sk} &= \frac{1}{2}(Q\partial_k C - C\partial_k Q) \qquad (2.6)\\
G_{kl} &= \frac{1-C}{2}(-\partial_k\partial_l \ln Q + \partial_k C\partial_l \ln Q + \partial_k C\partial_l \ln Q - C\partial_k \ln Q\partial_l \ln Q) \\
&\quad + \frac{1}{2}(\partial_k C\partial_l C - \partial_k\partial_l C)
\end{aligned}
$$

The vector field A^s_μ can be dualized by introducing a Lagrange multiplier term

$$
\mathcal{L}_{LM} = \frac{1}{48}\epsilon^{\mu\nu\lambda\rho\sigma}F^s_{\mu\nu}H_{\lambda\rho\sigma} \qquad (2.7)
$$

where $H_{\lambda\rho\sigma} = \partial_\lambda b_{\rho\sigma} + \partial_\sigma b_{\lambda\rho} + \partial_\rho b_{\sigma\lambda}$ is the field strength of the antisymmetric tensor field. The antisymmetric tensor field equations impose the Bianchi identity on F_s. The duality transformation is performed by eliminating F_s with the use of its equation of motion:

$$
2G_{ss}F_s + 2G_{sk}F_k - \frac{1}{4}Q_{kl}\,\epsilon F_k A_l - \frac{1}{12}\epsilon H = 0\ , \qquad (2.8)
$$

where $Q_{kl} \equiv \partial_k\partial_l Q$. After substituting F_s into eq.(2.2) the bosonic part of the lagrangian becomes

$$
\begin{aligned}
\mathcal{L}_{\mathrm{b}} &= -\frac{1}{2}\mathcal{R} - \frac{1}{2}g_{xy}\partial\phi^x\partial\phi^y + \frac{1}{192G_{ss}}H^2 - \frac{1}{4}(G_{kl} - \frac{G_{sk}G_{sl}}{G_{ss}})F_kF_l + \frac{1}{32G_{ss}}Q_{kl}HF_kA_l \\
&\quad - \frac{G_{sk}}{48G_{ss}}\epsilon HF_k + \frac{1}{48}(C_{klm} - \frac{3G_{sk}}{G_{ss}}Q_{lm})\epsilon F_kF_lA_m + \frac{3}{64G_{ss}}(Q_{kl}F_kA_l)^2 \qquad (2.9)
\end{aligned}
$$

As mentioned in the introduction, in the heterotic theory $1/s$ plays the role of the 5-D string coupling constant. The tree level metric of the gauge fields is then given by $G_{kl} = -\frac{1}{2}\partial_k\partial_l \ln Q$ and the only non-vanishing "Yukawa" couplings involve the antisymmetric tensor with two gauge fields. The couplings between three gauge bosons, as well as the mixing of gauge bosons with the antisymmetric tensor will appear at the one loop level.

3. Heterotic Superstring Compactified to $D = 5$ on $K_3 \times S_1$

$N = 2$ supersymmetric 5-D theory obtained by compactifying heterotic superstring on $K_3 \times S_1$ contains a gauge group with the rank ranging from 2 to 2+21, depending on details of compactification. The 2 gauge bosons are universal: the graviphoton and the vector dual to the antisymmetric tensor $b_{\mu\nu}$. Here we shall consider a rank 3 example with one additional $U(1)$ gauge boson associated to S_1. It can be constructed by following the lines of [15] and its further S_1 compactification to four dimensions yields the rank 4 model which on the type II side is described by $X_{24}(1,1,2,8,12)$ CY compactification [17, 9]. This model contains also 244 massless neutral hypermultiplets. We will derive the exact effective action describing the interactions of vector multiplets at the two-derivative level.

The vector moduli space of the above rank 3 model contains 2 real scalars, the dilaton ϕ whose expectation value determines the 5-D string coupling constant and the radius field R whose expectation value determines the radius of the circle S_1. At a generic point of this moduli space, the gauge group is $U(1)^3$, with the gauge bosons $b_{\mu\nu}$, $g_{\mu 6}$ and $b_{\mu 6}$, and there are no massless charged states. The massive Kaluza-Klein excitations and winding modes associated to S_1 are charged with respect to $g_{\mu 6}$ and $b_{\mu 6}$, but they are neutral with respect to $b_{\mu\nu}$. Their left and right momenta belong to a $O(1,1)$ lattice,

$$p_{R,L} = \frac{1}{\sqrt{2}}(\frac{m}{R} \pm nR) \; , \tag{3.1}$$

with integer m and n, therefore their masses depend on the radius R. At $R = 1$ two additional massless vector multiplets appear with $m = n = \pm 1$, so that $p_L = 0$ and $p_R = \pm\sqrt{2}$, and the $U(1)$ factor corresponding to the combination $A_\mu \sim g_{\mu 6} + b_{\mu 6}$ gets enhanced to $SU(2)$.

We will now derive the effective action by computing the appropriate superstring amplitudes. The string vertices for the gauge fields A and B corresponding to the right- and

left-moving combinations $g_{\mu 6} \pm b_{\mu 6}$, in the (-1)-ghost picture, are:

$$V_A^\mu(p,z) \;=\; :\psi^\mu \bar\partial X_6 e^{ip\cdot X}: \tag{3.2}$$

$$V_B^\mu(p,z) \;=\; :\psi_6 \bar\partial X^\mu e^{ip\cdot X}: \tag{3.3}$$

where X^μ and X_6 are the space-time and S_1 coordinates, respectively, while ψ^μ and ψ_6 are their world-sheet fermionic superpartners. The vertices for the graviton, dilaton (V_ϕ) and antisymmetric tensor are respectively the symmetric-traceless, trace and antisymmetric parts of

$$V^{\mu\nu}(p,z) =: \psi^\mu \bar\partial X^\nu e^{ip\cdot X}: \tag{3.4}$$

Finally, the vertex for the radius is:

$$V_R(p,z) =: \psi_6 \bar\partial X_6 e^{ip\cdot X}: \tag{3.5}$$

As explained in the introduction, the general form of the function $\mathcal{V}$ which determines the low energy effective action on the heterotic side is

$$\mathcal{V} = sQ(A,B) + C(A,B) \tag{3.6}$$

where the first term represents the tree level contribution depending on the quadratic form $Q(A,B)$ and the one loop correction is described by the cubic function $C(A,B)$. $Q(A,B)$ can be obtained from the tree-level three-point amplitudes involving one antisymmetric tensor and two gauge fields. It is easy to see that the only non-vanishing amplitudes are $\langle bAA \rangle$ and $\langle bBB \rangle$, giving

$$Q(A,B) = A^2 - B^2 \tag{3.7}$$

It follows then from eq.(2.9) that the gauge kinetic terms are not diagonal which can be confirmed by calculating the amplitude $\langle RAB \rangle$. Hence it is convenient to diagonalize the gauge kinetic terms by changing the vector field basis to

$$t_1 = A + B \qquad\qquad t_2 = B - A\ , \tag{3.8}$$

which is equivalent to going back to $g_{\mu 6}(=A_{1\mu})$ and $b_{\mu 6}(=-A_{2\mu})$. The scalar field surface $\mathcal{V}=1$ can be parameterized by

$$R=\left(\frac{t_1}{t_2}\right)^{1/2} \qquad\qquad \phi=\frac{2\pi}{t_1 t_2} \ . \tag{3.9}$$

In this field basis, $Q=t_1t_2$, and the the tree-level bosonic part of the lagrangian (2.9) becomes

$$\mathcal{L}_{\rm b}^{tree}=-\frac{1}{2}\mathcal{R}-\frac{\phi}{16\pi}(\frac{1}{R^2}F_1^2+R^2F_2^2)+\frac{\phi^2}{16\pi^2}(\frac{1}{24}H^2+\frac{1}{2}HF_1A_2)-\frac{1}{2}\frac{(\partial R)^2}{R^2}-\frac{3}{8}\frac{(\partial\phi)^2}{\phi^2} \tag{3.10}$$

The assignment of vertices V_ϕ and V_R to the scalars ϕ and R can be checked by computing three-point amplitudes involving one of these scalars and two gauge bosons. Indeed, using the vertices (3.2-3.5) one can show that the only non-vanishing amplitudes[4] of this type are $\langle\phi AA\rangle$, $\langle\phi BB\rangle$, $\langle RAB\rangle$ and $\langle\phi bb\rangle$, in agreement with the lagrangian (3.10) and the relations (3.8-3.9).

The one-loop function

$$C=a_1t_1^3+a_2t_2^3+b_1t_1^2t_2+b_2t_2^2t_1 \tag{3.11}$$

is parameterized by four constants $a_{1,2}$ and $b_{1,2}$. In fact, the physical amplitudes depend on $a_{1,2}$ only, since the last two terms can be eliminated by shifting the "dilaton" s,

$$s\rightarrow s-b_1t_1-b_2t_2 \ , \tag{3.12}$$

which corresponds to a perturbative symmetry of the heterotic theory. In order to extract $a_{1,2}$ it is sufficient to consider the "Yukawa" couplings between three gauge bosons. The relevant interaction terms obtained from eq.(2.9) have the form

$$\mathcal{L}_Y^{\textit{1-loop}}=\frac{1}{48}a_1\epsilon F_1F_1A_1+\frac{1}{8}(-2a_1R^2+\frac{a_2}{R^4})\epsilon F_1F_1A_2+(1\leftrightarrow 2, R\leftrightarrow\frac{1}{R}) \ . \tag{3.13}$$

[4] As usual, the space-time momenta have to be complexified in order to avoid kinematical constraints which make these amplitudes zero on-shell.

Using the above expression, it is straightforward to express all 3-gauge boson amplitudes in terms of the unknown constants $a_{1,2}$. In order to make contact with the superstring computation, it is convenient to go back to the A, B basis (3.8) of the vertex operators. One finds that the amplitudes $\langle BBB\rangle$ and $\langle BBA\rangle$ vanish while

$$\langle A_\mu(p_1)A_\nu(p_2)B_\lambda(p_3)\rangle \;=\; \frac{1}{2}\,\epsilon_{\mu\nu\lambda\rho\sigma}p_1^\rho p_2^\sigma(a_1R^3+\frac{a_2}{R^3}) \tag{3.14}$$

$$\langle A_\mu(p_1)A_\nu(p_2)A_\lambda(p_3)\rangle \;=\; \frac{3}{2}\,\epsilon_{\mu\nu\lambda\rho\sigma}p_1^\rho p_2^\sigma(a_1R^3-\frac{a_2}{R^3}) \tag{3.15}$$

The above one-loop amplitudes receive contribution from the odd spin structure only. There is one gauge boson vertex in the (-1)-ghost picture and the other two in the 0-picture which are obtained from (3.2,3.3) by replacing ψ^μ (ψ_6) by $\partial X^\mu+ip\cdot\psi\psi^\mu$ ($\partial X_6+ip\cdot\psi\psi_6$). In addition there is a world-sheet supercurrent insertion,

$$T_F =:\psi^\mu\partial X_\mu+\psi_6\partial X_6: +T_F^{\rm int} \tag{3.16}$$

where $T_F^{\rm int}$ represents the internal K_3 part, as well as the ghost contribution. In the odd spin structure, six fermionic zero-modes must be saturated to yield a non-zero result. It follows that the amplitudes $\langle BBB\rangle$ and $\langle BBA\rangle$ vanish as expected, while in the other two the saturation of the fermionic zero-modes gives rise to the ϵ-tensor as in eqs.(3.14,3.15). One obtains:

$$\langle A_\mu(p_1)A_\nu(p_2)B_\lambda(p_3)\rangle \;=\; \epsilon_{\mu\nu\alpha\rho\sigma}p_1^\rho p_2^\sigma\int_\Gamma\frac{d^2\tau}{\tau_2}\prod_{i=1}^{3}\int[d^2z_i] \\ \Big\langle\bar\partial X_6(\bar z_1)\bar\partial X_6(\bar z_2)\bar\partial X_\lambda(\bar z_3)\partial X^\alpha(0)\Big\rangle_{\rm odd} \tag{3.17}$$

$$\langle A_\mu(p_1)A_\nu(p_2)A_\lambda(p_3)\rangle \;=\; \epsilon_{\mu\nu\lambda\rho\sigma}p_1^\rho p_2^\sigma\int_\Gamma\frac{d^2\tau}{\tau_2}\int\prod_{i=1}^{3}[d^2z_i] \\ \Big\langle\bar\partial X_6(\bar z_1)\bar\partial X_6(\bar z_2)\bar\partial X_6(\bar z_3)\partial X_6(0)\Big\rangle_{\rm odd} \tag{3.18}$$

where $\tau=\tau_1+i\tau_2$ is the Teichmüller parameter of the world-sheet torus and Γ its fundamental domain.

The contraction of $\bar{\partial} X_\lambda$ with ∂X^α in eq.(3.17) gives $\left\langle \bar{\partial} X_\lambda(\bar{z}_3) \partial X^\alpha(0) \right\rangle = -\delta^\alpha_\lambda \pi / 4\tau_2$. The two $\bar{\partial} X_6$ insertions are replaced by their zero modes since their contractions yield total derivatives which vanish upon z_i integration. Similarly in eq.(3.18), ∂X_6 and all $\bar{\partial} X_6$'s are replaced by their zero modes. After performing the z_i integrations and taking into account the left-moving part of all determinants, we find:

$$\langle A_\mu(p_1) A_\nu(p_2) B_\lambda(p_3) \rangle = \frac{1}{2} \epsilon_{\mu\nu\lambda\rho\sigma} p_1^\rho p_2^\sigma \, \mathcal{I} \tag{3.19}$$

$$\langle A_\mu(p_1) A_\nu(p_2) A_\lambda(p_3) \rangle = \frac{3}{2} \epsilon_{\mu\nu\lambda\rho\sigma} p_1^\rho p_2^\sigma \, R \partial_R \mathcal{I} \tag{3.20}$$

where

$$\mathcal{I} = \frac{1}{16} \int_\Gamma \frac{d^2\tau}{\tau_2^2} \partial_{\bar{\tau}} (\tau_2^{1/2} Z) \bar{F}(\bar{\tau}) \tag{3.21}$$

with Z being the S_1 partition function,

$$Z = \sum_{p_L, p_R} e^{i\pi\tau p_L^2} e^{-i\pi\bar{\tau} p_R^2} \; . \tag{3.22}$$

$\bar{F}(\bar{\tau})$ is an R-independent antimeromorphic form of weight -2 in $\bar{\tau}$ with a simple pole at infinity due to the tachyon of the bosonic sector [13], $\bar{F} \sim \frac{i}{\pi^2} e^{-2i\pi\bar{\tau}}$. In deriving eq.(3.20) we used the explicit form of lattice momenta (3.1).

To evaluate the integral $\mathcal{I}$ we start from the identity:

$$[(R\partial_R)^2 - 1] Z = 16 \tau_2^{3/2} \partial_\tau \partial_{\bar{\tau}} (\tau_2^{1/2} Z) \tag{3.23}$$

which implies the following differential equation:

$$[(R\partial_R)^2 - 9] \mathcal{I} = \int_\Gamma d^2\tau \bar{F}(\bar{\tau}) \partial_\tau \{ \frac{1}{\tau_2^2} \partial_{\bar{\tau}} [\tau_2^2 \partial_{\bar{\tau}} (\tau_2^{1/2} Z)] \} \tag{3.24}$$

The r.h.s. being a total derivative with respect to τ vanishes away from the enhanced symmetric point $R = 1$. At $R = 1$, the surface term gives rise to a δ-function due to singularities associated to the additional massless particles which enhance the gauge symmetry to $SU(2) \times U(1)^2$. Expanding p_L, p_R around $R = 1$ for these states, it is easy to show that the surface term becomes proportional to:

$$\lim_{\tau_2 \to \infty} \tau_2^{1/2} e^{-\pi\tau_2 (R - \frac{1}{R})^2} = \frac{1}{2} \delta(R - 1) \; .$$

In terms of the "time" variable $\ln R$, eq.(3.24) becomes the one-dimensional propagator equation with the mass squared -9. Its general solution is

$$\mathcal{I} = \frac{1}{3} e^{-3|\ln R|} + \alpha(R^3 + \frac{1}{R^3}) \ , \tag{3.25}$$

where α is an arbitrary constant depending on the boundary conditions. Since at $R \to \infty$ $\mathcal{I} \to 0$ as $\mathcal{O}(R^{-3})$, *cf.* eq.(3.21), $\alpha = 0$, so that

$$\mathcal{I} = \frac{1}{3}[\theta(R-1)\frac{1}{R^3} + \theta(1-R)R^3] \ . \tag{3.26}$$

By comparing eqs.(3.19,3.20) with (3.14,3.15) we find

$$a_1 = \frac{1}{3}\theta(1-R)\,, \qquad\qquad a_2 = \frac{1}{3}\theta(R-1) \ . \tag{3.27}$$

This result can also be checked by studying the mixing between the antisymmetric tensor and the gauge bosons. The relevant lagrangian term, *cf.* eq.(2.9), is

$$\mathcal{L}_{HF}^{\textit{1-loop}} = \frac{1}{48}(-2a_1R^2 + \frac{a_2}{R^4})\epsilon H F_1 + (1 \leftrightarrow 2, R \leftrightarrow \frac{1}{R}) \ . \tag{3.28}$$

These interactions generate in particular the amplitudes involving one antisymmetric tensor field, one of the gauge bosons and one scalar. Going back to the A, B basis, it is easy to see that the only non-vanishing amplitudes involve the gauge field A and the radius scalar R:

$$\langle A_\mu(p_1) b_{\nu\lambda}(p_2) R(p_3)\rangle = \frac{1}{2}\,\epsilon_{\mu\nu\lambda\rho\sigma} p_1^\rho p_2^\sigma (a_1 R^3 + \frac{a_2}{R^3}) \tag{3.29}$$

The superstring computation of such amplitudes proceeds in a similar way as in the previous case. Only the odd spin structure contributes, and the zero-mode counting argument shows that (3.29) is indeed the only non-vanishing amplitude. One finds

$$\begin{aligned}\langle A_\mu(p_1) b_{\nu\lambda}(p_2) R(p_3)\rangle \;\;&=\;\; \epsilon_{\mu\nu\alpha\rho\sigma} p_1^\rho p_2^\sigma \int_\Gamma \frac{d^2\tau}{\tau_2} \prod_{i=1}^{3} \int [d^2 z_i] \\ &\qquad \left\langle \bar\partial X_6(\bar z_1) \bar\partial X_\lambda(\bar z_2) \bar\partial X_6(\bar z_3)) \partial X^\alpha(0) \right\rangle_{\rm odd} .\end{aligned} \tag{3.30}$$

This correlation function can be evaluated as (3.17) with the result

$$\langle A_\mu(p_1) b_{\nu\lambda}(p_2) R(p_3)\rangle = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho\sigma} p_1^\rho p_2^\sigma \,\mathcal{I} \tag{3.31}$$

which becomes compatible with the field-theoretical expression (3.29) after using eqs.(3.26) and (3.27).

To summarize, the result of the heterotic superstring computation is the function

$$\mathcal{V} = st_1t_2 + \frac{1}{3}\theta(t_2 - t_1)t_1^3 + \frac{1}{3}\theta(t_1 - t_2)t_2^3 \tag{3.32}$$

The discontinuity at $t_1 = t_2$ ($R = 1$) is due to the appearance of massless particles at the enhanced symmetry point. It is analogous to the logarithmic singularity of the prepotential in $D = 4$, however the infrared behavior of five-dimensional theory is different than in four dimensions.

In fact, eq.(3.32) can be derived by studying the 4-D theory obtained by a compactification on $K_3 \times T_2$ in the limit when one of the T_2 radii goes to infinity. In the rank 4 (STU) model of refs.[15, 9], the moduli space of T_2 is characterized by two complex moduli T and U which parameterize the metric and the antisymmetric tensor b_{56}. Taking $b_{56} = 0$ and a diagonal T_2 metric, with $g_{55} = R_5^2/2 \to \infty$ and $g_{66} = R^2/2$ we have:

$$T = iR_5\frac{R}{\sqrt{2\phi}} \qquad U = i\frac{R_5}{R\sqrt{2\phi}} \qquad S = 2iR_5\phi\ . \tag{3.33}$$

These equations follow from the usual relations between 5-D and 4-D scalars [7] upon the identification $t_1 \to T$, $t_2 \to U$ and $s \to S$ as dictated by the form of the lagrangian (3.10). Eqs.(3.33) imply that all three moduli go to infinity in the decompactification limit. The order of the these limits correspond to different domains of the (R, ϕ) space. The 5-D perturbative region corresponds to

$$(2\phi)^{-3/2} < R < (2\phi)^{3/2}\ , \tag{3.34}$$

which implies that the weak coupling limit $S \to i\infty$ has to be taken first. The tree-level "Yukawa" coupling $f_{STU} = 1$ coincides with C_{s12} in five dimensions upon the identification

$s = S$, $t_1 = T$, $t_2 = U$. Then, using the one loop result [13, 14][5]

$$f_{TTT} = -\frac{i}{\pi}\frac{j_T(T)}{j(T)-j(U)}\left\{\frac{j(U)}{j(T)}\right\}\left\{\frac{j_T(T)}{j_U(U)}\right\}\left\{\frac{j(U)-j(i)}{j(T)-j(i)}\right\} \tag{3.35}$$

we find that in the $R_5 \to \infty$ limit

$$f_{TTT} \to \frac{2}{1-e^{2\pi(R-1/R)R_5/\sqrt{2\phi}}} \to 2\theta(1-R) = 6a_1 = C_{111}\ . \tag{3.36}$$

Similarly,

$$f_{UUU} \to 2\theta(R-1) = 6a_2 = C_{222}\ . \tag{3.37}$$

The above expressions agree with the result (3.27) obtained by means of a direct computation in $D = 5$.

In four dimensions, the infinite moduli limit taken in the manner described above forces the effective theory into the perturbative regime, suppressing non-perturbative effects. This occurs, however, not only in the limit of asymptotically small 5-D coupling, but in the finite interval (3.34). Thus, the heterotic result (3.32) is exact in this region of the parameter space. $SU(2)$ gauge group remains unbroken at $R = 1$ at the non-perturbative level.

A similar analysis is even simpler for models of rank 2 which upon compactification to $D = 4$ become equivalent to type II superstring compactified in CY manifolds with $h_{(1,1)} = 3$, like $X_{12}(1,1,2,2,6)$ and $X_8(1,1,2,2,2)$ [15, 18]. In this case there is no enhanced gauge symmetry point and the heterotic tree-level action with $\mathcal{V} \sim st^2$ does not receive loop corrections in the weak coupling region $t < 1$.

4. Heterotic Superstring, 11-D Supergravity and p-branes

In this section we will discuss some aspects of duality between $N = 2$ supersymmetric heterotic superstring in five dimensions and 11-D supergravity theory compactified on a

[5] We make a factor 1/2 adjustment to the result [13] taking into account different normalization of S.

Calabi-Yau threefold. The latter contains BPS states obtained by wrapping two- and five-branes on even CY cycles. It has been argued before that 11-D supergravity describes the strong coupling limit of 10-D type IIA superstring theory [1]. In order to see a similar connection in $D = 5$, it is convenient to consider first further compactification from $D = 5$ to $D = 4$ on S_1. Then using duality between heterotic theory on $K_3 \times S_1 \times S_1$ and type II on Calabi-Yau, the 5-D decompactification limit $R_5 \to \infty$ corresponds to the large volume (complex structure) limit of the CY manifold on the type IIA (IIB) side, see eq.(3.33).

In the previous section we have shown that the effective action describing the heterotic side exhibits exact singularities due to enhanced gauge symmetries. These singularities, as viewed from the type II side, provide a strong evidence for the presence of enhanced gauge symmetry points on CY threefolds obtained by K_3 fibrations, in the large complex structure limit. This is similar to the case of K_3 twofolds where the presence of enhanced symmetries is related to duality in seven dimensions, between 11-D theory compactified on K_3 and heterotic superstring theory compactified on T_3 [1, 2, 19].

Let us first consider the central charge formula in five dimensions, for a generic supergravity theory. From the supersymmetry algebra it follows that the central charge is

$$Z_e = \sum_{\Lambda} t^{\Lambda} e_{\Lambda} \ , \tag{4.1}$$

where $t^{\Lambda} = (s, t^i)$ are the $D = 5$ special coordinates and e_{Λ} are the electric charges. The dual formula for the "magnetic" charges (string-like objects) is

$$Z_m = \sum_{\Lambda} t_{\Lambda} m^{\Lambda} \ , \tag{4.2}$$

where $\sum_{\Lambda} t^{\Lambda} t_{\Lambda} = 1$, therefore

$$t_{\Lambda} = C_{\Lambda\Sigma\Delta} t^{\Sigma} t^{\Delta} \ . \tag{4.3}$$

From the heterotic point of view e_i correspond to the usual (perturbative) electric charges of Kaluza-Klein excitations and winding modes, m^s is the charge of the fundamental string, while m^i and e_s arise from 10-D solitonic five-branes wrapping around K_3 and $K_3 \times S_1$,

respectively [20, 21, 22]. In the dual 11-D supergravity theory, these states originate from two- and five-brane solitons which wrap even cycles in the Calabi-Yau space [3, 16]:

$$e_\Lambda = \int_{\mathcal{C}^{4\Lambda}\times S_3} G_7 \ , \qquad\qquad m^\Lambda = \int_{\mathcal{C}_2^\Lambda\times S_2} F_4 \ , \tag{4.4}$$

where F_4 is the field strength of the three-index antisymmetric tensor field and $G_7 = \delta\mathcal{L}/\delta F_4$ is its dual; $\mathcal{C}_2^\Lambda$ and $\mathcal{C}_{4\Lambda}$, $\Lambda = 1,\ldots,h_{(1,1)}$, are two- and four-cycles in the CY space, respectively, while S_2 and S_3 are two- and three-dimensional spheres in 5-D spacetime.

In the case under discussion with two vector multiplets and $\mathcal{V} = \frac{1}{6}C_{\Lambda\Sigma\Delta}t^\Lambda t^\Sigma t^\Delta = sQ(t)+C(t)$, we have

$$Z_e \ = \ se_s + t^1e_1 + t^2e_2 \ = \ \frac{1-C(t)}{Q(t)}e_s + t^1e_1 + t^2e_2 \ . \tag{4.5}$$

For $C(t)=0$, this formula gives

$$Z_e = \frac{1}{g_5^2}e_s + g_5(Re_1 + \frac{1}{R}e_2) \ , \tag{4.6}$$

where $g_5^2 \equiv 2\pi/\phi$. Note that eq.(4.6) reproduces the $O(1,1)$ Narain lattice ($e_s = 0$) and also the Witten formula ($e_1 = e_2 = 0, e_s \neq 0$) [1] for the non-perturbative states which are electrically charged with respect to the $b_{\mu\nu}$ field. In the presence of one-loop corrections $C(t)$ calculated in the previous section, the central charge becomes:

$$Z_e = (\frac{1}{g_5^2} - \frac{1}{3}g_5R^3)e_s + g_5(Re_1 + \frac{1}{R}e_2) \ , \tag{4.7}$$

for $R<1$ and a similar expression with $R \to 1/R$ for $R>1$.

Eq.(4.7) raises the question about existence of massless states with vanishing Z_e (and/or Z_m). On the heterotic side, we do certainly have enhanced gauge symmetry at $R=1$ and possibly also at some other non-perturbative point or line with g_5 related to R. What is the interpretation of these points on the CY side? Considering 5-D CY theory as a decompactification limit of the 4-D theory corresponds to taking the limit $S \to i\infty$, $T \to i\infty$, $U \to i\infty$ while keeping all ratios fixed, see eq.(3.33). If we send $S \to i\infty$ first, we obtain the Yukawa coupling of ref.[17] in agreement with the perturbative heterotic

computation [13]. The result (3.36,3.37) exhibits a discontinuity due to the existence of an enhanced symmetry point at $T = U$. The other enhanced symmetry points, associated to $SU(3)$ and $SO(4)$ gauge groups, disappear for large T, U, however the $SU(2)$ gauge group remains intact at $T = U$ $(R = 1)$.

In the decompactification limit, the mass of a state as measured in $D = 5$ is related to its original 4-D mass in the following way:

$$M_5^2 = \lim_{R_5 \to \infty} R_5 \, M_4^2(R_5) \tag{4.8}$$

Furthermore, by comparing 5-D and 4-D theories it is easy to show that the respective moduli are related by [7]:

$$T^\Lambda = t^\Lambda R_5 \ , \tag{4.9}$$

where T^Λ and t^Λ are the 4-D and 5-D moduli, respectively, so that $C_{\Lambda\Sigma\Delta} T^\Lambda T^\Sigma T^\Delta = R_5^3$, in agreement with the standard supergravity result. Starting from the 4-D BPS mass formula [23] specified to the case of large S, T and U moduli, with the Kähler potential

$$K(S,T,U) \sim -\ln \mathcal{V} \sim -3 \ln R_5 \ , \tag{4.10}$$

one obtains (in the absence of magnetic charges)

$$M_5^2 = \lim_{R_5 \to \infty} R_5 \, M_4^2(R_5) = \lim_{R_5 \to \infty} e^K |S e_s + T e_t + U e_u|^2 R_5 = |s e_s + t e_t + u e_u|^2 \ , \tag{4.11}$$

in agreement with eq.(4.1).

The large radius limit is different however for states associated to singular points in the moduli space for which e^K does not fall off like R_5^{-3}. This happens for massive hypermultiplets which become massless at the conifold points [24]. Their 4-D mass is given by

$$M_4^2 = e^K |Z|^2 \ , \tag{4.12}$$

which goes to zero in the $Z \to 0$ conifold limit. Near $Z = 0$, the prepotential behaves as $\mathcal{F} \sim i Z^2 \ln Z$. The corresponding Kähler potential, after setting $Z = z R_5$, behaves as

$$e^{-K} = [2 z\bar{z} \ln z\bar{z} + 2 z\bar{z} \ln R_5^2 - (z - \bar{z})^2] R_5^2 \sim 2 z\bar{z} R_5^2 \ln R_5^2 \ . \tag{4.13}$$

Using eq.(4.12) we obtain

$$M_4^2 \sim \frac{1}{\ln R_5} \ , \tag{4.14}$$

so that

$$M_5^2 = \lim_{R_5 \to \infty} R_5 \, M_4^2 \sim \frac{R_5}{\ln R_5} \to \infty \ . \tag{4.15}$$

We see that although the 4-D mass goes to zero in the large radius limit, the 5-D mass diverges, therefore these states are not present in the decompactified theory. This is expected from the fact that they are due to the world-sheet instanton effects which arise from the mirror map, but such effects are not present in five dimensions as mentioned in section 2.

On the other hand, massive vector multiplets which never become massless in $D = 4$, keep a finite mass in the 5-D decompactification limit. Furthermore, two massless vector multiplets appear at $R = 1$, enhancing one of the $U(1)$'s to $SU(2)$. This can be compatible with the type IIA description if we accept the existence of enhanced symmetry points on Calabi-Yau threefolds in the large volume limit, for $t^1 = t^2$ *i.e.* at $R = 1$. A similar phenomenon occurs in the case of the heterotic superstring compactified on T_3 which is dual to 11-D theory compactified on K_3, where enhanced symmetry points do indeed exist [1, 2, 19]. As shown in refs.[1, 19], the enhanced symmetry points of the Narain lattice correspond to rational curves which shrink to zero size *i.e.* orbifold points on the K_3 side. A necessary condition for the existence of such points is the vanishing of the two-index antisymmetric tensor field. This is automatic in 11-D supergravity with p-branes since there is no two-form wrapping a complex curve. It follows that the dual pair consisting of 11-D theory compactified on K_3 and heterotic superstring compactified on T_3 is fully described by classical physics of the K_3 side. In the case of CY threefolds in $D = 11$ we are in a similar situation since there is no two-index antisymmetric tensor field, hence there are no "instanton" effects [3, 16]. Here again, massless vector multiplets do appear, reflecting the underlying K_3 fibration of the Calabi-Yau manifold. The description of our dual pair as a classical Calabi-Yau compactification of 11-D theory should remain exact. Note that

in a more general case, massless charged hypermultiplets could also exist at the enhanced symmetry points. Their vacuum expectation values would connect Calabi-Yau threefolds with distinct topologies in analogy with the 4-D example of ref.[10].

The central charge formulae (4.1-4.7) indicate that further enhanced symmetries (and/or massless states) may be present for other values of t^1, t^2.[6] In particular, massless states could appear which are charged with respect to $b_{\mu\nu}$. They would induce non-perturbative modifications of the Yukawa couplings, possibly generating $\mathcal{V}$-terms that are also quadratic and/or cubic in s. In the next section we will show that such points do indeed exist in the strong coupling regime of 5-D heterotic theory.

5. Strong Coupling Regime of 5-D Heterotic Theory

In order to derive the effective heterotic action in the strong coupling regime, we will first compactify the 5-D model to $D = 4$ on a circle of radius R_5. By using duality we know that the exact theory is described by type II superstring compactified on an appropriate CY threefold. Then, we go back to $D = 5$ by taking the limit $R_5 \to \infty$ in a way that corresponds to the strong coupling of the heterotic model. We will illustrate this procedure on the two-moduli (ST) example which on the type II side correspond to $X_{12}(1,1,2,2,6)$ CY model.

This model contains, at least in the weak coupling region, the antisymmetric tensor multiplet coupled to $N = 2$ supergravity in five dimensions.[7] The effective action (2.9) is described in this region exactly by the function

$$\mathcal{V} = st^2 + b\,t^3 \ , \tag{5.1}$$

where b is a constant. The t^3 term is unphysical at the perturbative level since it can be

[6] These could be related to non-perturbative enhanced symmetries recently discussed in ref.[25].

[7] It contains also 129 hypermultiplets which are irrelevant to the following discussion.

removed by shifting s as in eq.(3.12).

Eq.(5.1) can be recovered from the decompactification limit of the corresponding $D=4$ rank 3 model, with the 4-D moduli fields identified as

$$T = itR_5 \qquad\qquad S = i\frac{R_5}{t^2}\ . \tag{5.2}$$

The weak coupling region is defined by $\mathrm{Im}S > \mathrm{Im}T$ which corresponds to $t < 1$. This implies that when $R_5 \to \infty$ the limit $S \to i\infty$ should be taken first to recover the weakly coupled heterotic model in $D = 5$. In fact as $S \to i\infty$, the heterotic prepotential has the form

$$\mathcal{F} = ST^2 + f(T) + \mathcal{O}(e^{2i\pi S}) \tag{5.3}$$

where f is the one loop correction of ref.[26]. f is defined up to a quartic polynomial with real coefficients which can be removed by a symplectic change of basis that is also a symmetry of the perturbative theory. Furthermore, it is easy to show that $\partial_T^5 f \to 0$ as $T \to i\infty$. It follows that in the decompactification limit $\mathcal{V}$ is given by eq.(5.1) in the weak coupling region $t < 1$.

To find the decompactification limit in the full range of the coupling t, we consider the exact prepotential of the dual type II model $X_{12}(1,1,2,2,6)$ [15]. The Yukawa couplings are given by the following expression, as functions of the $N = 2$ special coordinates T_1 and T_2 [27, 17]:

$$\mathcal{F}_{ijk} = \mathcal{F}^0_{ijk} + \sum_{0\le n_1,n_2\in \mathbf{Z}} \frac{n_i n_j n_k N(n_1,n_2) q_1^{n_1} q_2^{n_2}}{1 - q_1^{n_1} q_2^{n_2}}\ , \tag{5.4}$$

where $q_i = \exp(2i\pi T_i)$ and $\mathcal{F}^0_{ijk}$ are the intersection numbers with $\mathcal{F}^0_{111} = 4$, $\mathcal{F}^0_{112} = 2$ and $\mathcal{F}^0_{122} = \mathcal{F}^0_{222} = 0$; $N(n_1,n_2)$ are the instanton numbers, the first few of which have been explicitly given in ref.[27]. In order to relate type II to its dual heterotic theory, the special type II coordinates T_1 and T_2 must be mapped to the special heterotic coordinates S and T. The perturbative tests of duality [15, 26] dictate $T_1 = T$ and $T_2 = S + \alpha T$, where α is an arbitrary constant. In ref.[9] it has been argued that $\alpha = -1$ based on the physical

requirement that the non-perturbative monodromy transformation $T_1 \to T_1 + T_2,\ T_2 \to -T_2$ preserves the positivity of ImS *i.e.* of the inverse square of the coupling constant.

Let us consider first the decompactification limit $S, T \to i\infty$ in the weak coupling region Im$S >$ ImT. In such a limit the instanton sum of eq.(5.4) vanishes and we obtain:

$$\mathcal{V} = st^2 - \frac{1}{3}t^3 \qquad\qquad (t < 1)\ , \tag{5.5}$$

in agreement with eq.(5.1) with $b = -1/3$. Note that the perturbative symmetry (3.12) is broken by non-perturbative effects to a quantized dilaton shift $s \to s + nt$ with n integer. Hence the t^3 term of eq.(5.5) cannot be removed at the non-perturbative level.

In the strong coupling region Im$T >$ Im$S \to \infty$, the instanton sum of eq.(5.4) becomes

$$-\sum_{0 \leq n_1 < n_2 \in \mathbf{Z}} n_i n_j n_k N(n_1, n_2) \tag{5.6}$$

For $n_1 \geq 1$ the instanton numbers $N(n_1, n_2)$ vanish for $n_1 < n_2$ while $N(0, n_2) = 2\delta_{1n_2}$ [27]. Hence, only $\mathcal{F}_{222}$ receives a non vanishing contribution -2 from the instanton sum. This result can also be obtained from the small q_1 expansion of the Yukawa couplings given in eq.(5.7) of ref.[28]:

$$\mathcal{F}_{222} = \frac{2q_2}{1 - q_2} + \mathcal{O}(q_1)\ . \tag{5.7}$$

It follows that

$$\mathcal{V} = s^2 t - \frac{1}{3}s^3 \qquad\qquad (t > 1)\ . \tag{5.8}$$

Note that eq.(5.8) is the same as eq.(5.5) with s and t interchanged reflecting a non-perturbative symmetry.

The final result for the function $\mathcal{V}$ in the two-moduli model, eqs.(5.5,5.8), can be written as

$$\mathcal{V} = [st^2 - \frac{1}{3}t^3]\theta(1 - t) + [s^2 t - \frac{1}{3}s^3]\theta(t - 1)\ . \tag{5.9}$$

As mentioned in section 3, this model has no enhanced gauge symmetry point in the weak coupling regime. The conifold singularity which is present in four dimensions at $T = i$

disappears upon decompactification to $D = 5$. On the other hand a discontinuity appears in the exact theory at $t = 1$ which corresponds in four dimensions to the non-perturbative singularity at $S = T$ ($q_2 = 1$, *cf.* eq.(5.7)). This is a fixed point of the non-perturbative monodromy transformation $S \leftrightarrow T$ which exchanges the heterotic string coupling constant with the compactification radius. Following the discussion of section 3, the singularity at $t = 1$ must be due to solitonic excitations which become massless at this point in five dimensions. Their existence is clearly a generic feature of the 5-D theory.

In fact, the above analysis can be extended in a straightforward way to the rank 3 model studied in section 3 which upon compactification to $D = 4$ becomes dual to type II superstring on $X_{24}(1, 1, 2, 8, 12)$ CY threefold. Here again, in the strong coupling region $(2\phi)^{-3/2} > R > (2\phi)^{3/2}$ [$\mathrm{Im}S < \max\{\mathrm{Im}T, \mathrm{Im}U\}$] one finds θ-function discontinuities which are due to non-perturbative states that become massless at $S = T$ and $S = U$.

In conclusion, there is a correspondence between the singularity structure of moduli spaces of 5-D and 4-D theories which comes out very clearly from our analysis. In $D = 4$, the enhanced symmetry points generically disappear for finite values of the heterotic coupling constant, being replaced by conifold singularities. In $D = 5$, enhanced symmetries survive non-perturbative effects and conifold singularities are absent. The additional singularities whose existence is a generic feature of CY compactifications have a very clear interpretation in $D = 5$. They are due to massless non-perturbative states which manifest their presence through discontinuities of the effective action. It would be interesting to determine what are the quantum numbers of these states.

Acknowledgments

We are grateful to R. Khuri, W. Lerche, P. Mayr and especially to A. Klemm for very useful conversations. I.A. thanks the Department of Physics at Northeastern University for its kind hospitality. S.F. and T.R.T. acknowledge the hospitality of Aspen Center for Physics during initial stage of this work.

References

[1] E. Witten, Nucl. Phys. B 443 (1995) 85.

[2] C.M. Hull and P.K. Townsend, Nucl. Phys. B 438 (1995) 109.

[3] A.C. Cadavid, A. Ceresole, R. D'Auria and S. Ferrara, Phys. Lett. B 357 (1995) 76.

[4] G. Papadopoulos and P.K. Townsend, Phys. Lett. B 357 (1995) 300.

[5] M.J. Duff, in: *Architecture of Fundamental Interactions at Short Distances,* Proceedings of 1985 Les Houches Summer School, P. Ramond and R. Stora, eds. (North Holland) p. 819.

[6] P.S. Howe and P.K. Townsend, in: *Proceedings of the Trieste Conference on Supermembranes and Physics in* $2+1$ *Dimensions*, M. Duff, C.N. Pope and E. Sezgin, eds. (World Scientific, 1990) p. 165.

[7] M. Günaydin, G. Sierra and P.K. Townsend, Nucl. Phys. B 242 (1984) 244; Nucl. Phys. B 253 (1985) 573.

[8] S. Cecotti, S. Ferrara and L. Girardello, Int. J. Mod. Phys. A4 (1989) 2475.

[9] A. Klemm, W. Lerche and P. Mayr, Phys. Lett. B 357 (1995) 313.

[10] P.S. Aspinwall, hep-th/9510142.

[11] M. Bershadsky, C. Vafa and V. Sadov, hep-th/9510225.

[12] B. de Wit and A. Van Proeyen, Phys. Lett. B 293 (1992) 94.

[13] I. Antoniadis, S. Ferrara, E. Gava, K.S. Narain and T.R. Taylor, Nucl. Phys. B 447 (1995) 35.

[14] B. de Wit, V. Kaplunovsky, J. Louis and D. Lüst, Nucl. Phys. B 451 (1995) 53.

[15] S. Kachru and C. Vafa, Nucl. Phys. B 450 (1995) 69.

[16] K. Becker, M. Becker and A. Strominger, hep-th/9507158.

[17] S. Hosono, A. Klemm, S. Theisen and S.-T. Yau, Comm. Math. Phys. 167 (1995) 301.

[18] I. Antoniadis and H. Partouche, hep-th/9509009.

[19] P.S. Aspinwall, Phys. Lett. B 357 (1995) 329.

[20] A. Strominger, Nucl. Phys. B 343 (1990) 167.

[21] M.J. Duff and R. Minasian, Nucl. Phys. B 436 (1995) 507; J.A. Dixon, M.J. Duff and J.C. Plefka, Phys. Rev. Lett. 69 (1992) 3009.

[22] J.A. Harvey and G. Moore, hep-th/9510182.

[23] A. Ceresole, R. D'Auria, S. Ferrara and A. Van Proeyen, Nucl. Phys. B 444 (1995) 92.

[24] A. Strominger, Nucl. Phys. B 451 (1995) 96; B.R. Greene, D.R. Morrison and A. Strominger, Nucl. Phys. B 451 (1995) 109.

[25] E. Witten, hep-th/9511030.

[26] I. Antoniadis, S. Ferrara, E. Gava, K.S. Narain and T.R. Taylor, in: *Proceedings of the Trieste conference on S-duality and Mirror Symmetry in String Theory, 5-9 June 1995*, hep-th/9510079; V. Kaplunovsky, J. Louis and S. Theisen, Phys. Lett. B 357 (1995) 71.

[27] P. Candelas, X. De la Ossa, A. Font, S. Katz and D. Morrison, Nucl. Phys. B 416 (1994) 481.

[28] B.H. Lian and S.-T. Yau, hep-th/9507153.